Chapter 10 - Electricity and Magnetism

Electric Charge and Coulomb's Law

Quantisation & Conservation: Charge is quantised ($q = ne$) and conserved in isolated systems.

Electron charge (e):$-1.602 \times 10^{-19} C$

Proton charge*:* $+1.602 \times 10^{-19} C$

Coulomb's Law (Vector Form): Describes the electrostatic force between two-point charges. unlike charges attract; like charges repel.

$$F_{12} = k \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

Where k (Coulomb constant) ≈ $8.99 \times 10^9 N \cdot m^2 C^{-2}$

Engineering Note: In calculations, we often use the permittivity of free space ϵ_0:

$k = \frac{1}{4\pi\epsilon_0}$ where $\epsilon_0 \approx 8.85 \times 10^{-12} F/m$

Electric Field and Electric Potential

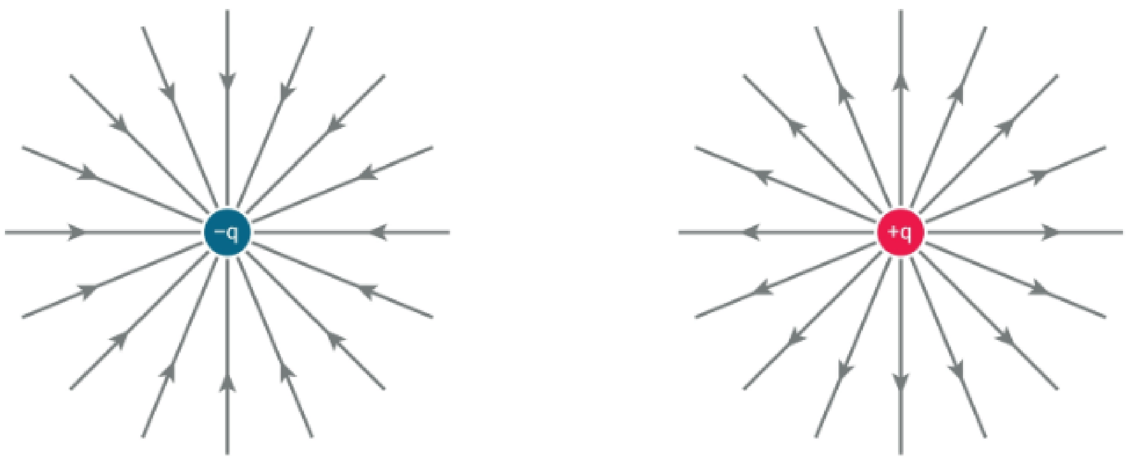

Figure 10.1: Visualization of Electric field lines due to charges

Electric Field (E): A vector field representing the force per unit positive charge at a point in space

$$E = \frac{F}{q_0}$$

Superposition Principle*:* The total E-field due to a distribution of charges is the vector sum of individual E-fields.

Electric Potential (V): A scalar quantity representing the electric potential energy (U) per unit charge. $V = \frac{U}{q_0}$ of $\Delta V = -\int_A^B \quad E \cdot dl$

Unit: Volts (V) or Joules/Coulomb (J/C).

Relation: The electric field points in the direction of the steepest decrease in potential ($E = -\nabla V$).

Equipotential Surfaces: Surfaces where V is constant. Electric field lines are always perpendicular to equipotential surfaces.

Magnetic Field and Magnetic Potential

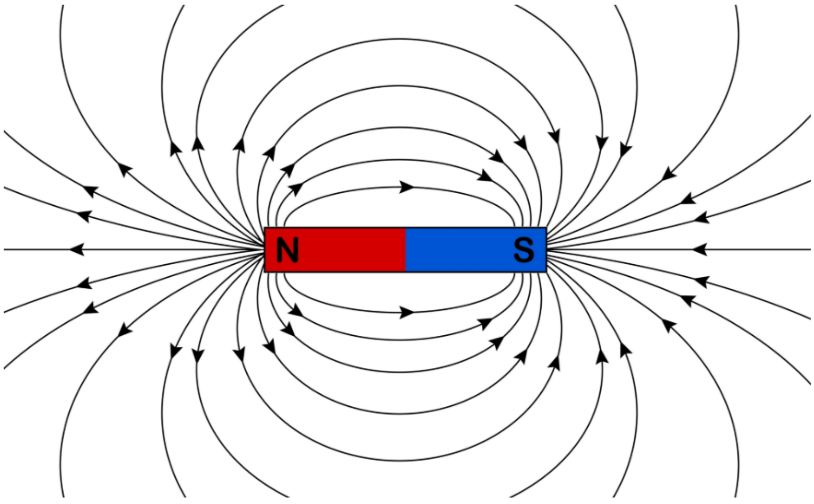

Figure 10.2: Magnetic field around a permanent magnet

Source of Magnetism: Unlike electric fields produced by static charges, Magnetic Fields (B) are produced by moving charges (currents) or intrinsic magnetic moments (spin).

The Biot-Savart Law: Calculates the B field produced by a current element Idl.

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \times \hat{r}}{r^2}$$

μ_0 (Permeability of free space) $= 4\pi \times 10^{-7} T \cdot m/A$

Direction: Determined by the Right-Hand Rule.

Comparison:

- **Electric:** Sources are monopoles (+ or -). Field lines start/end on charges.
- **Magnetic:** No magnetic monopoles exist. Field lines form closed loops.

Electromagnetic Force (Lorentz Force)

- **The Combined Force:** A charged particle moving with velocity v in the presence of both E and B fields experience the Lorentz force:

$$F = q(e + v \times B)$$

Magnetic Force Specifics:

- Force is maximum when velocity is perpendicular to the B-field ($\theta = 90°$).
- Force is zero if the particle moves parallel to the field lines.
- *Work:* Magnetic force does no work on the particle because F is always perpendicular to v. It changes the direction (centripetal force), not the speed.

Electrical Circuits

Ohm's Law and Resistance

Macroscopic Form: The current through a conductor is proportional to the voltage across it. $V = IR$

Resistivity (ρ): An intrinsic material property. Resistance R is determined by geometry: $R = \rho \frac{L}{A}$

Temperature Dependence: For metals, resistivity increases with temperature: $\rho = \rho_0[1 + \alpha(T - T_0)]$

Kirchhoff's Laws (Circuit Analysis)

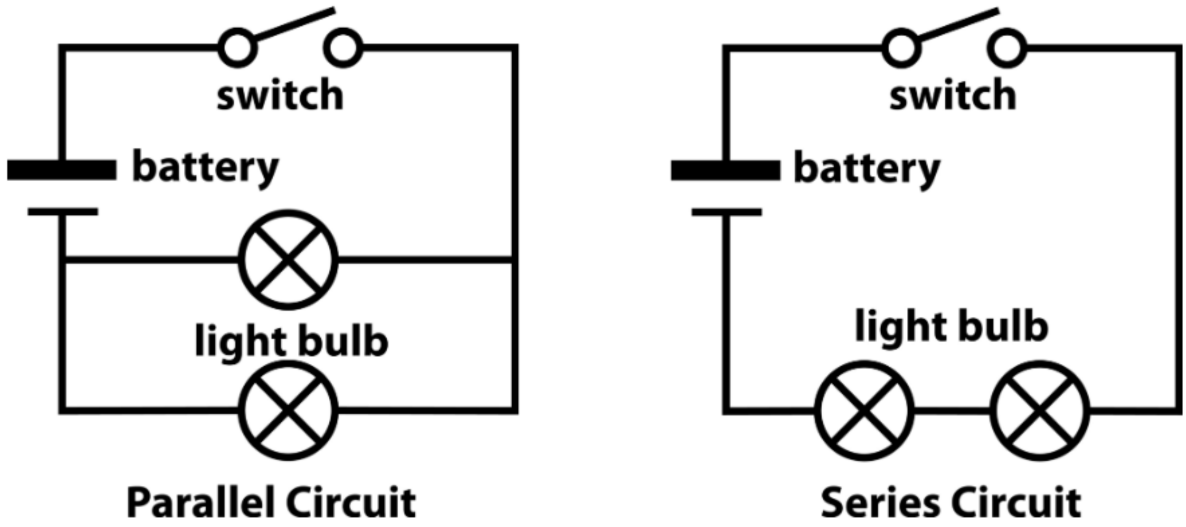

Figure 10.3: circuit types

Kirchhoff's Current Law (KCL): Based on **Conservation of Charge**

$\sum I_{in} = \sum I_{out}$ at any junction(node).

Kirchhoff's Voltage Law (KVL): Based on **Conservation of Energy**

$\sum \Delta V = 0$ around any closed loop

Convention: Voltage drops across a resistor in the direction of current ($-IR$); voltage rises going from negative to positive terminal of a battery ($+E$)

Series and Parallel Configurations

Feature	Series Circuits	Parallel Circuits
Current	Same through all components ($I_{total} = I_1 = I_2$)	Splits across branches ($I_{total} = I_1 + I_2$))
Voltage	Splits across components ($V_{total} = V_1 + V_2$)	Same across all components $V_{total} = V_1 = V_2$)
Equivalent Resistance	$R_{eq} = R_1 + R_2 + \cdots R_n.$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$
Failure Mode	One break stops all current (Open Circuit).	One break affects only that branch.

Table 1: Table showing comparison between series and parallel circuits

DC vs. AC Circuits

Direct Current (DC): Unidirectional flow. Source: Batteries, Solar cells.

Alternating Current (AC): Current reverses direction periodically (Sinusoidal). Source: Wall outlets, Generators.

- *Key Metric:* **RMS (Root Mean Square)** values for AC voltage and current are used to calculate equivalent power.
- *AC* allows easy voltage stepping (via transformers) for efficient long-distance transmission

Electromagnetic Induction

Magnetic Flux

- Defined as the amount of magnetic field passing through a surface area A.

$$\Phi_B = \int B \cdot dA = BAcos\theta$$

Unit: Weber (Wb)

Faraday's Law of Induction

A changing magnetic flux induces an electromotive force (EMF,E) in a loop

$$E = -N\frac{d\Phi_B}{dt}$$

This is the operating principle behind **generators**, **transformers**, and **induction cooking**

Lenz's Law

The "Negative" Sign: The direction of the induced current is such that it creates a magnetic field to **oppose the change** in magnetic flux that produced it.

Example: If you push a magnet into a coil, the coil creates a field pushing back. This ensures Conservation of Energy.

Generators and Motors

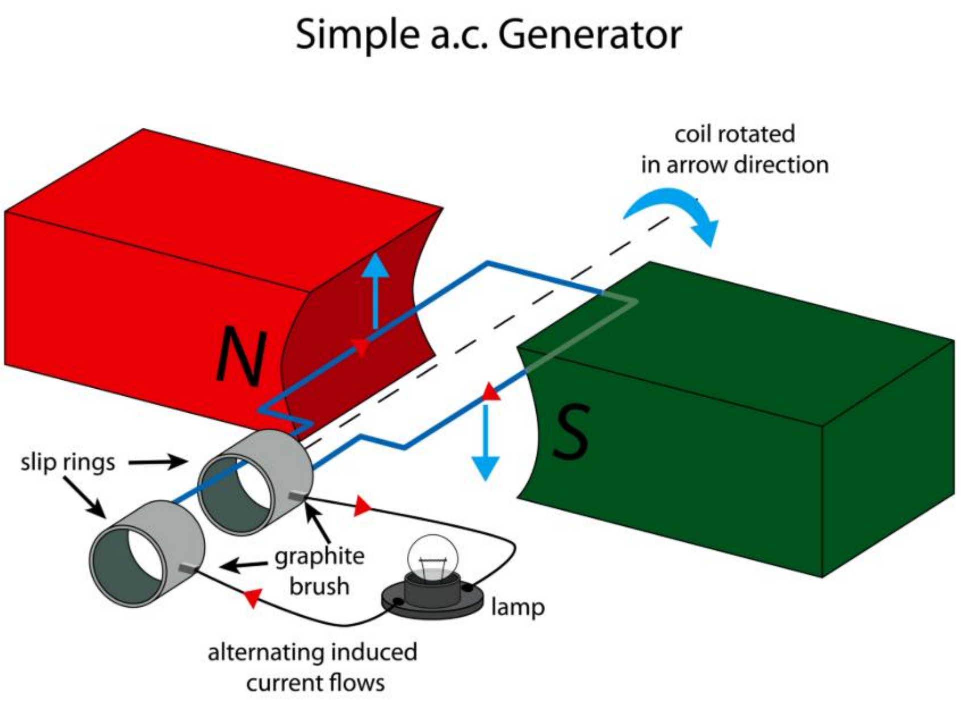

Figure 10.4: Simple AC generator

Generator: Mechanical Energy →Electrical Energy.

- Rotate a coil in a B-field →Change Φ_B → Induce Current.

Motor: Electrical Energy →Mechanical Energy.

- Current in a coil within a B-field →Lorentz Force creates Torque →Rotation

Activities & Practical Applications

1. Laboratory Practical: CUMSM 110 No.3 (Intro to Components & Safety)

- **Focus:** Multimeter usage (Ammeter in series, Voltmeter in parallel).
- **Safety Note:** High current is more dangerous than static high voltage. Always ensure power is off before modifying a circuit.

2. Laboratory Practical: CUMSM 110 No.4 (Series & Parallel)

- **Objective:** Verify R_{eq} formulas experimentally.
- **Troubleshooting:** If readings don't match theory, check for loose connections (added resistance) or blown fuses in the multimeter.

3. Project Prototype (Applicable)

- *Idea:* Build a simple DC motor using a battery, magnet, and copper wire.
- *Idea:* Construct a small solenoid and measure the magnetic field strength vs. current.

Exercises

1. A particle with a charge of $1.1\ nC$ and a mass of $1.2 \times 10^{-9} kg$ moves with velocity $\vec{v} = (1.7 \times 10^4 ms^{-1})\hat{\imath} + (1.8 \times 10^4 ms^{-1})\hat{\jmath}$ as it enters a uniform magnetic field of $\vec{B} = (1.2\ T)\hat{\imath}$.

   a) Calculate the magnetic force acting on the particle as it enters the field.

   b) Describe the path of the particle, and determine its radius.
   c) Calculate the time it takes for the particle to complete one revolution.
   d) Calculate the distance the particle travels along the magnetic field lines as it completes one revolution.

2. Electric eels generate electric pulses along their skin that can be used to stun an enemy when they encounter it. Research has shown that these pulses can be up to $500\ V$ and produce currents of $80\ mA$ (or even larger). A typical pulse lasts for $10\ ms$. Calculate:

   a) The charge released,
   b) the power and,
   c) how much energy are delivered to the unfortunate enemy with a single pulse, assuming a steady current.

3. A straight wire of length 20 cm carries a current of 5 A. The wire is placed in a uniform magnetic field of 0.2 T that is perpendicular to the wire. Find the magnetic force on the wire and the magnetic flux through a rectangular loop of area $0.04\ m^2$ that encloses the wire.
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