Chapter 9: Electrostatics

Chapter Overview

Electrostatics is the analysis of electromagnetic phenomena that occur when there are no moving charges (static equilibrium). It forms the foundation for all electromagnetic theory (Maxwell's Equations). For engineers, understanding electrostatics is crucial not just for circuit design, but for analysing stress in dielectric materials, designing sensors and actuators (MEMS), and understanding the fundamental limits of insulation in high-voltage power systems.

9.1 Electric Charge and Its Properties

Fundamental Definition: Charge (q) is an intrinsic property of matter. All matter is made of atoms containing **positively charged protons**, **negatively charged electrons**, and **neutral neutrons**.
When the number of protons and electrons in an object differ, it becomes charged.

- **Like charges** repel each other.
- **Opposite charges** attract each other.

The **SI unit of charge** is the **coulomb (C)**.
An electron carries a charge of -1.6×10^{-19} C, while a proton carries $+1.6 \times 10^{-19}$ C.

Key properties:

- **Quantization of charge:** Electric charge exists in discrete amounts (multiples of 1.6×10^{-19} C).
- **Conservation of charge:** Charge can neither be created nor destroyed; it can only be transferred. In an isolated system, the algebraic sum of charges remains constant.
- **Charging Mechanisms:**
    - **Conduction:** Transfer via physical contact.
    - **Induction:** Redistributing charge without contact (crucial for grounding and shielding).

Example: When you rub a balloon against your hair, electrons move from your hair to the balloon.

Your hair becomes positively charged, while the balloon becomes negatively charged — a classic case of **charge transfer** through friction.

9.2 Coulomb's Law and the Electric Field

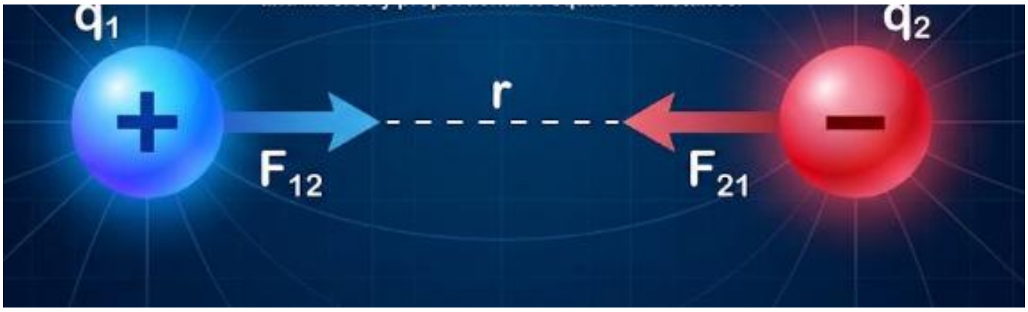

Figure 9.1: Point charges

Coulomb's law quantifies the force between two-point charges:

$F_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$ where:

- F = electrostatic force (N)
- q_1, q_2 = magnitudes of the charges (C)
- r = distance between the charges (m)
- $k = 8.99 \times 10^9$ N·m²/C² (Coulomb's constant)

The force acts along the line joining the charges **repulsive** for like charges and **attractive** for opposite ones.

Electric Field (E)

The **electric field** is the region around a charge where another charge experiences a force.

It is defined as the force per unit charge: $\mathrm{E} = \frac{F_o}{q_o}$

- For a continuous distribution, we integrate: $\mathrm{E} = k \int \frac{dq}{r^2} \hat{r}$
- The field is a **vector quantity**.

- **Field lines** originate from positive charges and terminate at negative charges.
- The **density** of the lines represents field strength.

Electric Dipoles: Two equal and opposite charges separated by a distance d. Important for understanding dielectric materials.

- **Dipole Moment (p):** $p = qd$.
- **Torque on a Dipole:** $\tau = p \times E$

Engineering Connection: Electric fields are used to control electron beams in **CRT displays**, charge particles in **photocopiers**, and apply coatings in **electrostatic painting**.

Gauss's Law

Electric Flux (Φ_E): A measure of the number of field lines passing through a surface A.

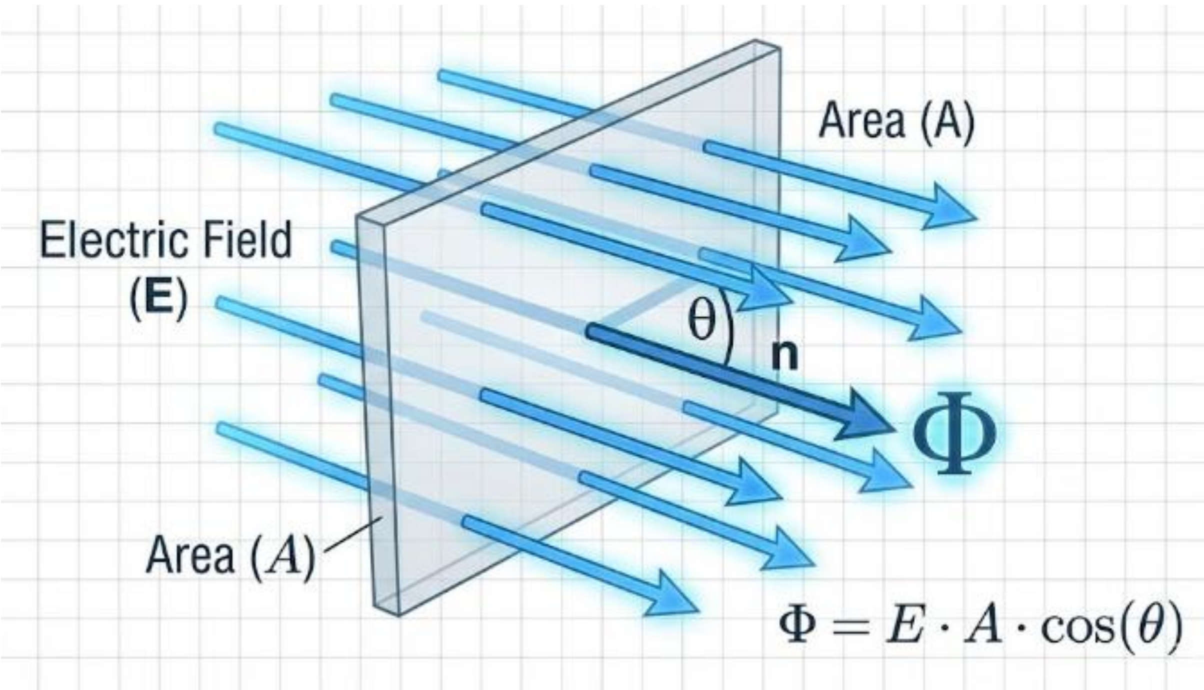

Figure 9.2: electric flux passing through a surface

$$\Phi_E = \int E \cdot dA$$

Gauss's Law Statement: The net flux through any closed surface (Gaussian surface) is proportional to the enclosed charge (Q_{enc}).

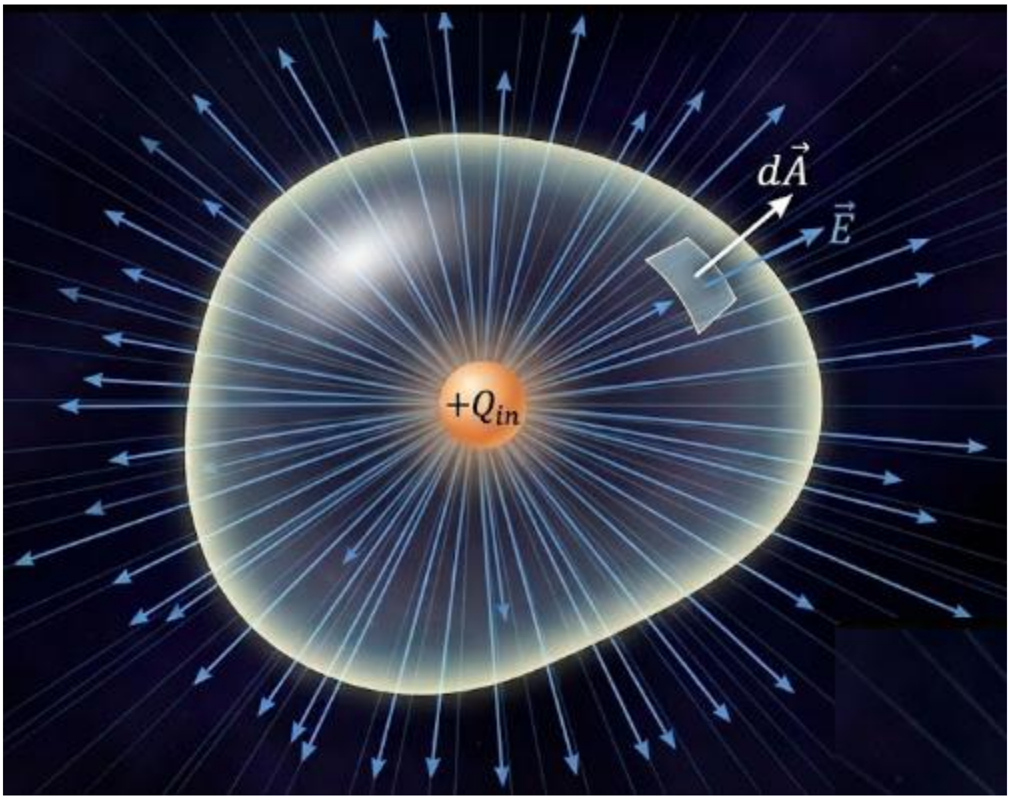

Figure 9.3: Gaussian surface

$$\oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

Engineering Application: Used to easily calculate fields for symmetric systems (coaxial cables, spherical shells, infinite sheets).

Electric Potential and Energy

Electric Potential (V): The work done per unit charge to bring a test charge from infinity to a point P. $V = -\int_{\infty}^{P} E \cdot ds$

Relationship to Field: E is the negative gradient of V.

$$E = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Electric Potential Energy (U): The energy required to assemble a system of charges. $U = k\sum\frac{q_i q_j}{r_{ij}}$

Electron Volt (eV): A common unit in semiconductor engineering $1eV = 1.602 \times 10^{-19}J$

Capacitance and Dielectrics

Capacitance (C): Defined largely by geometry. $C = \frac{Q}{\Delta V}$

Common Geometries:

Parallel Plate: $c = \frac{\epsilon_0 A}{d}$

Energy Storage: The potential energy stored in the electric field of a capacitor:

$$U_E = \frac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

Dielectrics:

Insulators placed between plates increase capacitance by a factor κ (dielectric constant).

- **Mechanism:** The external field polarizes the dielectric atoms, creating an internal "reverse" field that reduces the net field, allowing more charge to be stored at the same voltage.
- **Dielectric Strength:** The maximum field a material can withstand before breakdown (arcing) occurs.

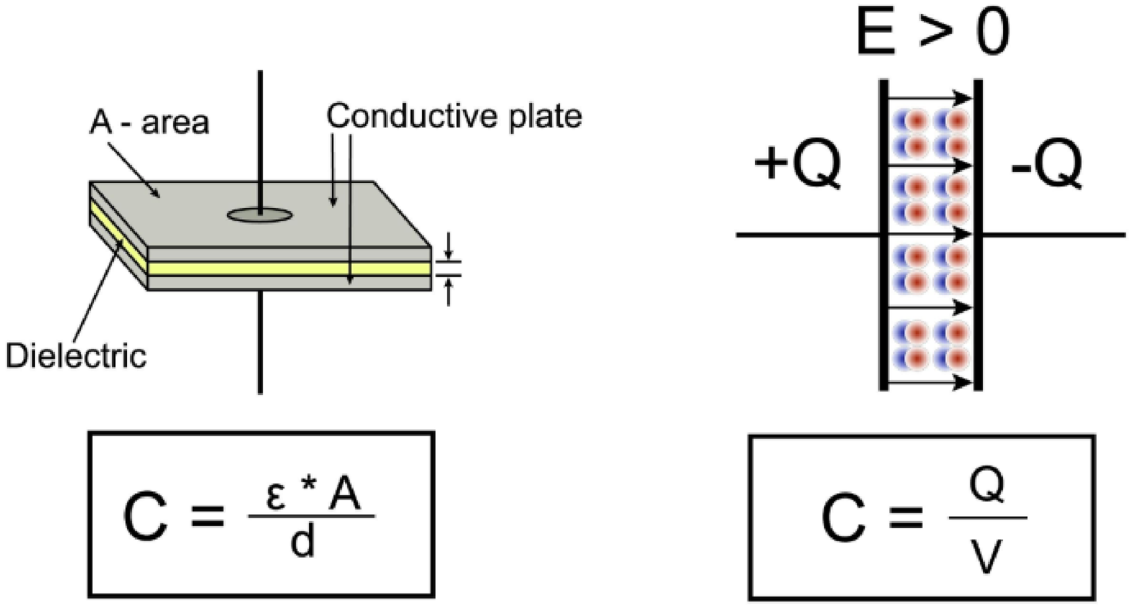

Figure 9.4: Parallel plate capacitor

Conductors in Electrostatic Equilibrium

Understanding conductors is vital for safety and circuit shielding.

1. **Zero Internal Field:** The electric field inside a perfect conductor is zero.
2. **Surface Charge:** All net charge resides on the surface.
3. **Perpendicular Field:** The electric field just outside a charged conductor is perpendicular to the surface.
4. **Sharp Points:** Charge density (σ) is highest at points with the smallest radius of curvature (basis for lightning rods).

Engineering Applications

- **The Cathode Ray Tube (CRT):** Deflection of electron beams using E-fields (legacy displays, oscilloscopes).
- **Electrostatic Precipitators:** Industrial pollution control using corona discharge to ionize smoke particles and attract them to collection plates.

- **Xerography (Photocopiers):** Using a photoconductive drum (selenium) to map an optical image into a static charge distribution.
- **MEMS (Micro-Electro-Mechanical Systems):** Using electrostatic forces to move tiny mirrors (in projectors) or sense acceleration (in airbag sensors).
- **Breakdown Voltage:** Designing insulators for high-voltage transmission lines to prevent "flashover" based on the dielectric strength of air.

Concept Summary

- Electric charge is a fundamental property of matter, existing in positive and negative forms.
- Coulomb’s law governs the force between static charges.
- Electric fields describe how forces act in space, while electric potential describes energy changes.
- Capacitors store charge and energy; dielectrics enhance their performance.
- Conductors and semiconductors are essential to modern electronics.
- Electrostatics is the foundation of many technologies — from printers and air purifiers to smartphones and solar cells.

Exercise

1. What is meant by charge quantization?
2. Two charges of +2 μC and –3 μC are separated by 0.5 m. Calculate the force between them.
3. Explain why no work is done moving a charge along an equipotential surface.
4. How does inserting a dielectric affect the capacitance of a capacitor?
5. Give two examples of semiconductor applications that rely on electrostatic control.
6. A $+25.0\ nC$ charge is placed at the origin of a coordinate system, and a $-15.0\ nC$ charge is placed in the xy-plane at the point$(x = 0.200\ m, y = -0.100\ m)$.

a) Determine the magnitude and the direction of the force that the positive charge exerts on the negative charge. Use ijk notation for your result.
b) Determine the magnitude and direction of the force of the negative charge on the positive charge.

7. A capacitor that has two parallel plates of dimensions $15.00cm \times 25.00\ cm$ separated by $3.00\ mm$ of air is connected to a $12.0\ V$ battery. Find
   a) the capacitance
   b) the charge on one plate
   c) the electric field in the capacitor