Chapter 3: Work and Energy

3.1 Introduction to Work and Energy

Work and energy are fundamental concepts in physics and engineering. Work describes the transfer of energy when a force is applied to an object causing displacement, while energy represents the capacity to do work. These two concepts are closely related and underpin many engineering systems, from machines and engines to electrical devices and construction processes.

Understanding work and energy enables engineers to analyse, design, and optimize processes for efficiency and sustainability.

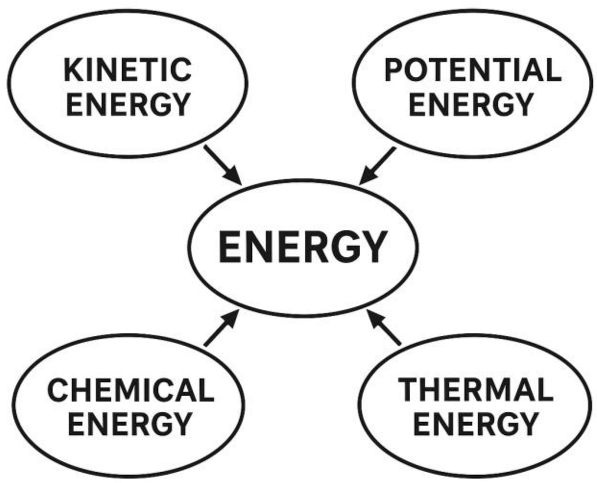

Figure 3.1: Relationship between different forms of energy.

3.2 What is Work?

In physics, work is done when a force is applied to an object and the object is displaced in the direction of the force.

$$W = \vec{F} \cdot \vec{d} = Fd \cos\theta$$

Where:

W = Work done (Joules, J)

F = Magnitude of the force (N)

d = Displacement of the object (m)

θ = Angle between force and displacement

Key Notes:

Work is positive if the force has a component in the direction of displacement.

Work is negative if the force opposes displacement (e.g., friction).

If displacement is zero, no work is done.

Engineering Example:

A crane lifting a 200 kg steel beam vertically by 5 m does positive work against gravity:

$$W = Fd = (200 \times 9.81)(5) = 9810\, J$$

3.3 What is Energy?

Energy is defined as the capacity to do work. It exists in different forms such as kinetic, potential, thermal, chemical, and electrical.

The law of conservation of energy states that energy cannot be created or destroyed, only transformed from one form to another.

Unit of energy: Joule (J)

1 Joule = Work done when a force of 1 N displaces an object by 1 m in the direction of the force.

3.4 The Work-Energy Theorem

The work done on an object is equal to the change in its kinetic energy:

$$W = \Delta KE = KE_f - KE_i$$

Example:

A car of mass 1000 kg accelerates from rest to 20 m/s.

$$W = \frac{1}{2} mv^2 = \frac{1}{2}(1000)(20^2) = 200\,000\,J$$

3.5 Types of Energy

Energy appears in multiple forms:

1. Kinetic Energy (KE): Energy due to motion

$$KE = ½ mv^2$$

2. Potential Energy (PE): Energy stored due to position or configuration

Gravitational PE : $PE = mgh$,

Elastic PE $PE = ½ kx^2$

3. Thermal Energy: Due to particle motion (heat).
4. Electrical Energy: Work done by moving electric charges.
5. Chemical Energy: Stored in bonds, released during reactions.
6. Nuclear Energy: Released from atomic nuclei (fission/fusion).

3.6 Calculating Work and Energy

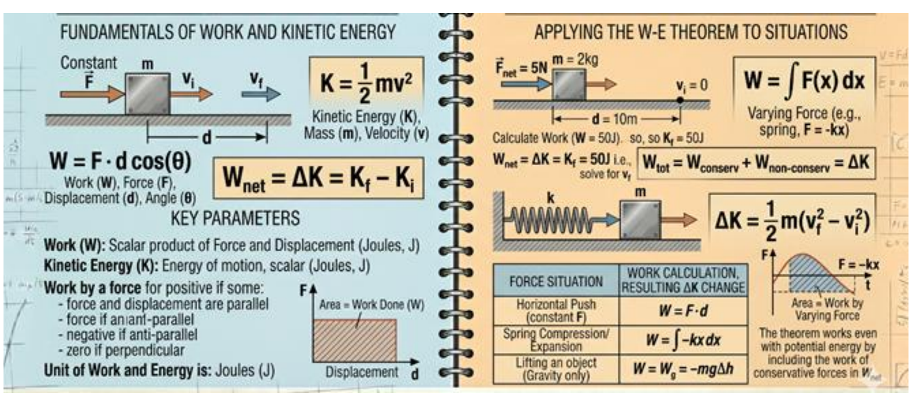

Figure 3.2: Illustration of the work and energy theorem

Work Done by a Constant Force

$$W = Fd\cos\theta$$

Example:

Pushing a cart with 50 N force over 10 m at 30°

$$W = 433\,J$$

Work Done by a Variable Force

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

Example:

Work done by a spring

$$W = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

Work Done by Friction2

$W = -f_k d,$

Where $f_k = \mu_k N,$ the kinetic friction force

Example:

10 kg block slides 5 m on a surface with μ_k = 0.2

$f_k = \mu_k mg = 0.2 \times 10 \times 9.81 = 19.62N$

$$W = 19.62 \times 5 = -98.1\,JW = mgh$$

3.7 Energy Calculations and Applications

- Engineering Mechanics: Calculating lifting power of cranes.
- Automotive: Work-energy theorem used in braking systems.
- Renewable Energy: Energy conversion efficiency in wind turbines.
- Civil Engineering: Potential energy considerations in dam design.

3.8 Applications of the Work-Energy Theorem

- Design of shock absorbers (dissipation of KE).
- Determining required motor power in conveyor systems.
- Estimating braking distances of vehicles.
- Analysing roller coasters and structural loads.

3.9 Solving Engineering Problems Involving Work and Energy

Example Problem:
A 5 kg block slides down a frictionless incline of 4 m height. Find velocity at the bottom.

$$PE_{top} = KE_{bottom}$$

$$mgh = \frac{1}{2}mv^2$$

$$5 \times 9.81 \times 4 = \frac{1}{2} \times 5v^2$$

$$v = \sqrt{(2gh)}$$

$$v = \sqrt{78.48} = 8.85\, m/s$$

3.10 Summary

- Work is the product of force and displacement in the direction of force.
- Energy is the capacity to do work, existing in many forms.
- The work-energy theorem connects force, motion, and energy.
- Different forces (gravity, friction, variable forces) influence work differently.
- Applications in engineering are vast, from machines to infrastructure.

Exercises

1. A 1000 kg car accelerates from 10 m/s to 25 m/s. Calculate the work done on the car.
2. A spring with k = 500 N/m is compressed by 0.1 m. Find the potential energy stored in the spring.
3. A block of 20 kg is pulled 10 m with a force of 80 N at 30°. Calculate the work done.
4. A 50 kg person climbs a staircase of height 3 m. Find the work done against gravity.
5. Explain how the work-energy theorem is applied in the design of car braking systems.