Chapter 2: Kinematics and Forces

Chapter Overview

- Kinematics and Forces are two fundamental topics in physics that deal with the motion and interaction of objects.
- Kinematics describes how objects move, such as their position, velocity, acceleration, and trajectory.
- Forces explain why objects move, such as the effects of gravity, friction, tension, and pressure.
- By applying the principles of kinematics and forces, we can analyse and predict the behaviour of various physical systems, such as projectiles, pendulums, springs, and collisions.

Unit Objectives

By the end of this unit, students should be able to:

- Describe and analyse motion in one and two dimensions, using vectors and scalar quantities.
- Apply Newton's laws of motion to solve problems involving forces.
- Understand the concepts of work, energy, and momentum.
- Apply the concepts of kinematics and forces to engineering problems.

2.1 Introduction

Kinematics and forces are central pillars of physics and engineering.

- **Kinematics** is the study of motion without considering its causes. It describes how objects move in terms of displacement, velocity, and acceleration.
- **Dynamics**, through the concept of forces, explains *why* objects move the way they do. Forces such as gravity, friction, tension, and applied pushes/pulls govern the motion of real-world systems.

Together, kinematics and forces allow engineers to design machines, predict projectile trajectories, analyse vehicle motion, study structures under load, and even understand planetary orbits.

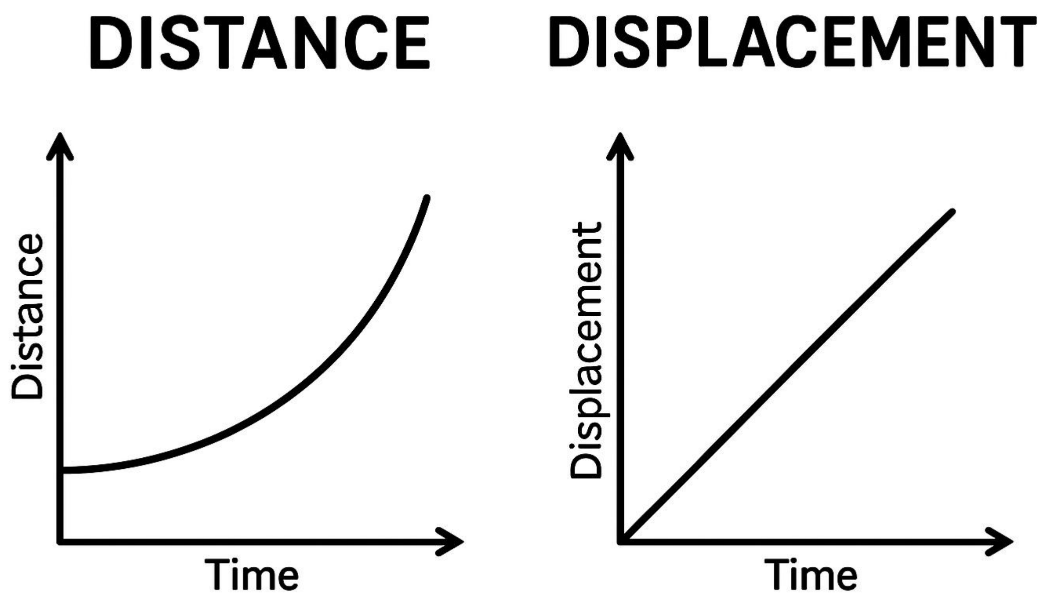
2.2 Kinematics in One Dimension

2.2.1 Displacement and Distance

Distance (d): A scalar quantity that measures the total length of the path travelled, regardless of direction.

Displacement (Δx): A vector quantity defined as the change in position:

$$\Delta x = x_f - x_i$$



The **distance-time graph** always increases with time since distance is a scalar quantity that only measures the total path covered, regardless of direction. The curve indicates that the object is accelerating.
The **displacement-time graph**, on the other hand, can increase, decrease, or stay constant depending on the motion, since displacement is a vector quantity that measures the shortest change in position. The straight line here shows uniform motion in one direction.

2.2.2 Speed and Velocity

Speed (v): The rate of change of distance (scalar).

$$v = \frac{d}{t}$$

Velocity (v): The rate of change of displacement (vector).

$$v = \frac{\Delta x}{\Delta t}$$

It can be positive or negative, depending on direction.

2.2.3 Acceleration

Defined as the rate of change of velocity:

$$a = \frac{\Delta v}{\Delta t}$$

Positive acceleration: Object speeds up.

Negative acceleration (deceleration): Object slows down.

2.2.4 Equations of Motion in 1D

For constant acceleration, the following equations apply:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Where.

u = initial velocity

v = final velocity

a = acceleration

s = displacement

t = time

2.2.5 Free Fall Motion

In free fall, only gravity acts (a=g=9.81 ms^2 near Earth).

- Velocity after time t:

$$v = gt$$

- Displacement after time t:

$$s = \frac{1}{2}gt^2$$

Worked Example 2.1

A stone is dropped from rest from a 50 m building. Find:
(a) The time to reach the ground.
(b) The velocity on impact.

Solution:

$$s = \frac{1}{2}gt^2 \Rightarrow 50 = \frac{1}{2}(9.81)t^2$$

$$t = \sqrt{\frac{100}{9.81}} = 3.19s$$

$$v = gt = 9.81(3.19) \approx 31{,}3\,ms^{-1}$$

Answer: Time = $3.19s$, Velocity = $31,3\,ms^{-1}$ (downward).

2.3 Kinematics in Two Dimensions

- Many real-world motions occur in a plane, requiring vector decomposition into **x- and y-components**.

2.3.1 Motion in a Plane

Position vector:

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

Velocity vector:

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = v_x\hat{\imath} + v_y\hat{\jmath}$$

Acceleration vector:

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = a_x\hat{\imath} + a_y\hat{\jmath}$$

2.3.2 Projectile Motion

Projectile motion combines uniform horizontal motion and vertical free fall.

Horizontal motion (no acceleration):

$$x = vxt$$

Vertical motion (accelerated by gravity):

$$y = v_y t - \frac{1}{2}gt^2$$

Trajectory equation:

$$y = x\tan\,\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$$

where θ is launch angle, v_0 is initial velocity.

Worked Example 2.2

A football is kicked with speed $20\,ms^{-1}$ at 30°∘ above the ground. Find:

(a) Time of flight

(b) Maximum height

(c) Horizontal range

Solution:

$$vx = v0\cos\cos 30\,^\circ = 20\times 0.866 = 17.32\,\frac{m}{s}$$

$$vy = v0\sin\,30\,^\circ = 20\times 0.5 = 10\,m/s$$

(a) **Time of flight:**

$$T = \frac{2v_y}{g} = \frac{20}{9.81} = 2.04\,s$$

(b) **Max height:**

$$H = \frac{v_y^2}{2g} = \frac{100}{19.62} = 5.1\,m$$

(c) **Range:**

$$R = v_xT = 17.32(2.04) = 35.3\,m$$

✅ The ball travels $35.3\,m$ in $2.04\,s$, reaching $5.1\,m$ high.

2.4 Newton's Laws of Motion

2.4.1 First Law – Inertia

An object continues in uniform motion unless acted upon by a net force.

Engineering Example: A train requires brakes (external force) to stop despite its large inertia.

2.4.2 Second Law – Acceleration

Force is proportional to acceleration:

$$\vec{F} = m\vec{a}$$

Example: Doubling the mass of a car requires doubling the engine force to achieve the same acceleration.

2.4.3 Third Law – Action-Reaction

For every action, there is an equal and opposite reaction.

Example: A rocket lifts off because the expelled gases push downward, while the rocket experiences an upward thrust.

Worked Example 2.4

A 1200 kg car accelerates from rest to $20\, m/s$ in $10\, s$. Find the net force.

$$a = \frac{v-u}{t} = \frac{20}{10} = 2\, ms^{-2}$$

$$F = ma = 1200(2) = 2400\, N$$

Required net force = 2400 N.

2.5 Work, Energy, and Momentum

2.5.1 Work

Work is the transfer of energy when a force is applied over a displacement.

The general formula is:

$$W = \vec{F} \cdot \vec{d} = Fd\theta$$

where θ is the angle between the force and displacement vectors.

👉 **Key Notes:**

Work is **positive** if the force has a component in the direction of displacement (e.g., pushing a cart forward).

Work is **negative** if the force opposes the motion (e.g., friction).

Work is **zero** if the force is perpendicular to displacement (e.g., centripetal force in circular motion).

Engineering Application:

Calculating the work done by engines, cranes, or pumps to move loads.

2.5.2 Energy

Energy is the capacity to do work. It exists in many forms, but two fundamental mechanical forms are:

2.5.3 Kinetic Energy

$$KE = \frac{1}{2}mv^2$$

Depends on mass and velocity.

Doubling velocity **quadruples** kinetic energy → critical in vehicle crash analysis.

2.5.4 Potential Energy

Gravitational Potential Energy

$$PE = mgh$$

where h is height above reference point.

Elastic Potential Energy: Stored in springs and elastic bodies:

$$PE_{elastic} = \frac{1}{2}kx^2$$

where k is the spring constant, x is extension/compression.

2.5.5 The Work-Energy Theorem

The net work done on a body is equal to the change in its kinetic energy:

$$Wnet = \Delta KE = KE_f - KE_i$$

👉 This theorem connects **forces** directly to **motion**.

Example: Braking a car converts kinetic energy into heat (via friction).

2.5.6 Momentum

Momentum measures the “quantity of motion” an object has:

$$p = mv$$

Conservation of Momentum

In an isolated system with no external forces:

$$m_1v_1i + m_2v_2i = m_1v_1f + m_2v_2f$$

👉 Critical for analysing **collisions, rocket propulsion, and explosions**.

2.5.7 Impulse and Momentum

Impulse is the product of force and time:

$$J = F\Delta t = \Delta p$$

- Large forces over short times cause big momentum changes (e.g., hammer strikes).
- Safety devices (airbags, cushioned barriers) **increase time of impact** → reduce force ex p erienced.

2.5.8 Connecting Work, Energy, and Momentum

F orce × displacement = Work → Energy

F orce × time = Impulse → Momentum

These two principles are like *two sides of the same coin*. One relates to how forces act over distances, the other to how they act over time.

2.5.9 Engineering Applications

1. **Cranes & Lifting Machines** – Work and potential energy calculations ensure motors are powerful enough.
2. **Automobiles** – KE and momentum determine braking distance, crash safety, and fuel efficiency.
3. **Sports Engineering** – Energy and momentum transfer in kicking a ball, hitting a golf shot, or designing protective gear.
4. **Rocket Propulsion** – Momentum conservation explains thrust: expelling gases at high speed propels the rocket forward.
5. **Industrial Machines** – Flywheels store rotational kinetic energy to smooth out energy supply.

Worked Example 2.4

A 2 kg block is pushed with a force of 10 N over 5 m horizontally. Calculate the work done.

$$W = \theta = 10 \times 5 \times \cos\cos 0° = 50 J$$

✅ Work = 50 J.

Exercise

1. A 10 kg block is pushed up a smooth incline 5 m long at an angle of $30°$ with a constant force of 60 N.
   a) Calculate the work done by the applied force.
   b) Find the gain in gravitational potential energy of the block.
   c) Explain why the two results differ or agree.
2. A 1200 kg car is moving at 25 m/s.
   a) Calculate its kinetic energy.
   b) If the driver brakes and the car stops in 50 m, find the average retarding force.
3. A spring with constant k=400 N/m is compressed by 0.1 m.
   a) Calculate the elastic potential energy stored.
   b) If the spring launches a 2 kg block horizontally, find the velocity of the block (neglecting friction).
4. A 0.2 kg ball moving at 10 m/s strikes a stationary 0.3 kg ball in a perfectly elastic collision.
   a) Find the velocities of both balls after impact.
   b) Verify conservation of momentum.
   c) Verify conservation of kinetic energy.
5. During a car crash, a passenger of mass 70 kg is brought from 20 m/s to rest.
   a) Calculate the change in momentum.
   b) If this happens in 0.1 s (without an airbag), find the average force.
   c) If an airbag increases the time to 0.5 s, find the new average force.
   d) Discuss the role of airbags using your results.